

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Equivalence of Damping from Flight Flutter Test Evaluation and Eigenvalue Calculation

Valter J. E. Stark*

Saab Scania AB, Linköping, Sweden

and

Helmut Wittmeyer†

Consultant, Linköping, Sweden

DERIVATION of general equations of motion for an elastic finite wing in terms of appropriate indicial aerodynamic coefficients, which has been done by Stark,^{1,2,3} and solution by Laplace transformation, which has been used by Stark^{4,5} and Edwards,⁶ yields an eigenvalue problem, which implies that the complex frequency shall be determined such that the determinant of the transformed equations becomes zero. The real part of this frequency or eigenvalue determines the true damping which we consider in this Note.

Due to the analytic property of the unsteady aerodynamic coefficients which result from the transformation, the results obtained for the eigenvalues in the way mentioned must agree with the results obtained by evaluation of response data from a flight flutter test by an accurate method. Using the method of Wittmeyer⁷ and fictitious test data, it is shown in this Note that the results indeed agree.

The method of Wittmeyer,⁷ which is also useful for ground vibration tests, has been found to give accurate results in many simulated flight flutter tests as well as in an evaluation of an actual flight flutter test of a large transport aircraft. It is especially designed for the case of neighboring frequencies and employs simultaneously the responses (deflections) $z_i(\omega)$ at n structural points. For a circular frequency ω near a resonance, one can write approximately:

$$z(\omega) = [z_1(\omega), \dots, z_n(\omega)]^T = e_1 / (p_1 - i\omega) + \dots + e_n / (p_n - i\omega) + c$$

where e_1, \dots, e_n are the eigenvectors that predominate in the response p_1, \dots, p_n , corresponding eigenvalues, and c a constant vector. If the structural points are not chosen unsuitably, there exists a vector v such that

$$v^T z(\omega) - 1 / (p_k - i\omega) - a_k = 0$$

where $a_k = v^T c$. The n unknown components of v , p_k , and a_k in this equation are determined by requiring that the equation

Table 1 Deflection modes

Mode	Frequency, Hz	
1	4.39	Wing bending
2	10.50	Wing-body-stabilizer-fin bending
3	11.50	Wing-body-stabilizer-opposite fin bending
4	14.34	Second order wing-body bending
5	15.64	Stabilizer bending

shall be satisfied (in the least squares sense) for, in general, more than $n+2$ values of ω near the resonance mentioned above. There appears only one nonlinear unknown, namely p_k , in this method, and only this is solved. For finding the eigenvectors, a special procedure is described in Ref. 7.

The fictitious flight test data which have been used in this Note represent structural deflections of a commuter aircraft due to gust excitation with harmonic variation in the flight direction. The data were obtained by solving five equations, which were obtained by Laplace transformation of general equations of motion in the time domain. They correspond to five measured modes.

The same equations were used in the eigenvalue calculation and the same analytic expression was used for approximation of aerodynamic coefficients in both cases. This is necessary for achieving agreement, but the accuracy of the approximation does not influence the agreement. The expression used is defined by Eq. (37) in Ref. 3 for $k_1 = k_2 = 3$ and $a = 5.5$.

In the eigenvalue calculation, which was performed by the first author, the simple and well-known Newton method was used for finding the zeros of the determinant. Based on this method, a simple and generally applicable program that avoids the "hunting problem" of Hassig's method⁸ has been developed.³ In this program, the determinant is calculated by triangularization and Gaussian elimination. By using a small increment in speed, iteration can virtually be avoided in practice.

In the evaluation of the fictitious flight flutter test, which was performed by the second author, the response at four structural points, located at the wing tip, the stabilizer tip, the stabilizer root, and the aircraft c.g. was used and only eight frequency values were utilized for each eigenvalue. For the flutter critical mode (Mode 3), these frequencies were not more than 1% different from the resonance frequency (for Mode 1 not more than 18%).

The mass ratio μ for the outboard part of the wing was about 16. The deflection modes considered, which were determined in a ground vibration test, are slightly unsymmetric. They are described in Table 1.

The results from the two evaluations are presented in Table 2 for a few Mach numbers. By comparison it is seen that the results are very close indeed, as they should be.

Finally, it may be mentioned that Wittmeyer⁹ has made a similar comparison by using aerodynamic coefficients given only at discrete points on the imaginary axis (calculated for a two-dimensional incompressible flow in the comparison). When solving the flutter equations, he assumes, like Stark,³ analyticity of the aerodynamic coefficients and expands the eigenvalue in a Taylor series. This procedure also produced

Received May 19, 1983. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1983. All rights reserved.

*Research Scientist. Member AIAA.

†Before retirement, Head of Flutter and Vibration, Saab Scania AB.

Table 2 Comparison of eigenvalues

Mode No.	Mach No.	Eigenvalue calculation		Flight test evaluation	
		ζ	ω/ω_r	ζ	ω/ω_r
3	0.30	0.00780	2.4831	0.00778	2.4832
3	0.32	0.00638	2.4737	0.00637	2.4737
3	0.34	0.00471	2.4645	0.00471	2.4645
3	0.36	0.00294	2.4559	0.00294	2.4559
3	0.38	0.00129	2.4480	0.00129	2.4480
1	0.38	0.1472	0.9689	0.1466	0.9794

Note: $p = \sigma + i\omega$ = complex frequency $\zeta = -\sigma/|p|$ ω_r corresponds to $f_r = 4.93$ Hz

closely agreeing results. In addition, Wittmeyer⁹ made a comparison with Hassig's p-k method⁸ and found that the p-k method predicted damping about 14% too high.

References

- ¹ Stark, V.J.E., "The Flutter Problem," *Saab Report*, FKLF-0-76:46, Linköping, Sweden, 1976 (In Swedish).
- ² Stark, V.J.E., "Manual for AEREL—A Program System for Aeroelastic Analysis," *Saab Report* FKLF-0-78:25, Linköping, Sweden, April 1978.
- ³ Stark, V.J.E., "General Equations of Motion for an Elastic Wing and Method of Solution," *AIAA*, Paper 83-0921, May 1983.
- ⁴ Stark, V.J.E., "Flutter Calculation for the Viggen Aircraft with Allowance for Leading Edge Vortex Effect," *AGARD Conference: Unsteady Airloads in Separated and Transonic Flow*, April 1977, C.P. No 226.

⁵ Stark, V.J.E., "Test of Pines' Approximate Method in a Flutter Calculation for the Viggen Aircraft," *Journal of Aircraft*, Vol. 14, No. 7, July 1977, pp. 702-703.

⁶ Edwards, J.W., "Unsteady Aerodynamic Modelling for Arbitrary Motions," *AIAA Journal*, Vol. 15, No. 4, April, 1977, pp. 593-595.

⁷ Wittmeyer, H., "Parameter Identification for Structures with Neighbouring Natural Frequencies Especially in Case of Flight Resonance Tests," *Z. Flugwiss. Weltraumforsch.* Vol. 6, No. 2, 1982, pp. 80-90. (In German)

⁸ Hassig, H.J., "An Approximate True Damping Solution of the Flutter Equation by Determinant Iteration," *Journal of Aircraft*, Vol. 8, No. 11, Nov. 1971, pp. 885-889.

⁹ Wittmeyer, H., "Calculation of Subcritical Natural Frequencies and Damping Rates in a Flutter Investigation," To be published in *Z. Flugwiss. Weltraumforsch.*, Vol. 7, No. 5, 1983, pp. 331-334.

From the AIAA Progress in Astronautics and Aeronautics Series . . .

VISCOUS FLOW DRAG REDUCTION—v. 72

Edited by Gary R. Hough, Vought Advanced Technology Center

One of the most important goals of modern fluid dynamics is the achievement of high speed flight with the least possible expenditure of fuel. Under today's conditions of high fuel costs, the emphasis on energy conservation and on fuel economy has become especially important in civil air transportation. An important path toward these goals lies in the direction of drag reduction, the theme of this book. Historically, the reduction of drag has been achieved by means of better understanding and better control of the boundary layer, including the separation region and the wake of the body. In recent years it has become apparent that, together with the fluid-mechanical approach, it is important to understand the physics of fluids at the smallest dimensions, in fact, at the molecular level. More and more, physicists are joining with fluid dynamicists in the quest for understanding of such phenomena as the origins of turbulence and the nature of fluid-surface interaction. In the field of underwater motion, this has led to extensive study of the role of high molecular weight additives in reducing skin friction and in controlling boundary layer transition, with beneficial effects on the drag of submerged bodies. This entire range of topics is covered by the papers in this volume, offering the aerodynamicist and the hydrodynamicist new basic knowledge of the phenomena to be mastered in order to reduce the drag of a vehicle.

456 pp., 6 × 9, illus., \$25.00 Mem., \$40.00 List

TO ORDER WRITE: Publications Order Dept., AIAA, 1633 Broadway, New York, N.Y. 10019