# Engineering Notes

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## Equivalence of Damping from Flight Flutter Test Evaluation and Eigenvalue Calculation

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DERIVATION of general equations of motion for an elastic finite wing in terms of appropriate indicial aerodynamic coefficients, which has been done by Stark,[1,2,3] and solution by Laplace transformation, which has been used by Stark[4,5] and Edwards,[6] yields an eigenvalue problem, which implies that the complex frequency shall be determined such that the determinant of the transformed equations becomes zero. The real part of this frequency or eigenvalue determines the true damping which we consider in this Note.

Due to the analytic property of the unsteady aerodynamic coefficients which result from the transformation, the results obtained for the eigenvalues in the way mentioned must agree with the results obtained by evaluation of response data from a flight flutter test by an accurate method. Using the method of Wittmeyer[7] and fictitious test data, it is shown in this Note that the results indeed agree.

The method of Wittmeyer,[7] which is also useful for ground vibration tests, has been found to give accurate results in many simulated flight flutter tests as well as in an evaluation of an actual flight flutter test of a large transport aircraft. It is especially designed for the case of neighboring frequencies and employs simultaneously the responses (deflections) $z_i(\omega)$ at $n$ structural points. For a circular frequency $\omega$ near a resonance, one can write approximately:

$$z(\omega) = [z_1(\omega),...,z_n(\omega)]^T = \boldsymbol{e}_1/(p_1 - i\omega) + ...$$
$$+ \boldsymbol{e}_n/(p_n - i\omega) + \boldsymbol{c}$$

where $\boldsymbol{e}_1,...,\boldsymbol{e}_n$ are the eigenvectors that predominate in the response $p_1...,p_n$, corresponding eigenvalues, and $\boldsymbol{c}$ a constant vector. If the structural points are not chosen unsuitably, there exists a vector $\boldsymbol{v}$ such that

$$v^T z(\omega) - 1/(p_k - i\omega) - a_k = 0$$

where $a_k = v^T \boldsymbol{c}$. The $n$ unknown components of $\boldsymbol{v}$, $p_k$, and $a_k$ in this equation are determined by requiring that the equation

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Table 1 Deflection modes

| Mode | Frequency, Hz | |
|---|---|---|
| 1 | 4.39 | Wing bending |
| 2 | 10.50 | Wing-body-stabilizer-fin bending |
| 3 | 11.50 | Wing-body-stabilizer-opposite fin bending |
| 4 | 14.34 | Second order wing-body bending |
| 5 | 15.64 | Stabilizer bending |

shall be satisfied (in the least squares sense) for, in general, more than $n+2$ values of $\omega$ near the resonance mentioned above. There appears only one nonlinear unknown, namely $p_k$, in this method, and only this is solved. For finding the eigenvectors, a special procedure is described in Ref. 7.

The fictitious flight test data which have been used in this Note represent structural deflections of a commuter aircraft due to gust excitation with harmonic variation in the flight direction. The data were obtained by solving five equations, which were obtained by Laplace transformation of general equations of motion in the time domain. They correspond to five measured modes.

The same equations were used in the eigenvalue calculation and the same analytic expression was used for approximation of aerodynamic coefficients in both cases. This is necessary for achieving agreement, but the accuracy of the approximation does not influence the agreement. The expression used is defined by Eq. (37) in Ref. 3 for $k_i = k_s = 3$ and $a = 5.5$.

In the eigenvalue calculation, which was performed by the first author, the simple and well-known Newton method was used for finding the zeros of the determininant. Based on this method, a simple and generally applicable program that avoids the "hunting problem" of Hassig's method[8] has been developed.[3] In this program, the determinant is calculated by triangularization and Gausian elimination. By using a small increment in speed, iteration can virtually be avoided in practice.

In the evaluation of the fictitious flight flutter test, which was performed by the second author, the response at four structural points, located at the wing tip, the stabilizer tip, the stabilizer root, and the aircraft c.g. was used and only eight frequency values were utilized for each eigenvalue. For the flutter critical mode (Mode 3), these frequencies were not more than 1% different from the resonance frequency (for Mode 1 not more than 18%).

The mass ratio $\mu$ for the outboard part of the wing was about 16. The deflection modes considered, which were determined in a ground vibration test, are slightly unsymmetric. They are described in Table 1.

The results from the two evaluations are presented in Table 2 for a few Mach numbers. By comparison it is seen that the results are very close indeed, as they should be.

Finally, it may be mentioned that Wittmeyer[9] has made a similar comparison by using aerodynamic coefficients given only at discrete points on the imaginary axis (calculated for a two-dimensional incompressible flow in the comparison). When solving the flutter equations, he assumes, like Stark,[3] analyticity of the aerodynamic coefficients and expands the eigenvalue in a Taylor series. This procedure also produced

Table 2 Comparison of eigenvalues

| Mode No. | Mach No. | Eigenvalue calculation | | Flight test evaluation | |
|---|---|---|---|---|---|
| | | $\zeta$ | $\omega/\omega_r$ | $\zeta$ | $\omega/\omega_r$ |
| 3 | 0.30 | 0.00780 | 2.4831 | 0.00778 | 2.4832 |
| 3 | 0.32 | 0.00638 | 2.4737 | 0.00637 | 2.4737 |
| 3 | 0.34 | 0.00471 | 2.4645 | 0.00471 | 2.4645 |
| 3 | 0.36 | 0.00294 | 2.4559 | 0.00294 | 2.4559 |
| 3 | 0.38 | 0.00129 | 2.4480 | 0.00129 | 2.4480 |
| 1 | 0.38 | 0.1472 | 0.9689 | 0.1466 | 0.9794 |

Note: $p=\sigma+i\omega$ = complex frequency $\zeta=-\sigma/|p|$ $\omega_r$ corresponds to $f_r=4.93$ Hz

closely agreeing results. In addition, Wittmeyer[9] made a comparison with Hassig's p-k method[8] and found that the p-k method predicted damping about 14% too high.

## References


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